Design Study of a Ring Stiffened Cylinder for use as a Manned Submersible

Shell Buckling using Von Mises Equation - Widenberg, D.F and Trilling, C.
"Collapse by Instability of aThin Cylindrical Shells Under External Pressure"
ASME Trans, Volume 56, 1934, P.820, Equation [6]

$SafetyFactor := 2.0$

$DesignGoal := 1320 \cdot ft \cdot SafetyFactor$ $\qquad DesignGoal = 2640\,ft$

Design Variables:

Outside Diameter	$OD := 42.0 \cdot in$
Shell Thickness	$t := .375 \cdot in, .4375 \cdot in .. .625 \cdot in$
Shell Length	$Len := 104.25 \cdot in$
Number of Rings	$num := 2$

Constants:

$SeaWaterDensity := 64 \cdot \frac{lbf}{ft^3}$

Material Properties:

Poissons Ratio	$\mu := .3$
Yield Strength	$\sigma := 38000 \cdot \frac{lbf}{in^2}$
Youngs Modulus	$E := 30 \cdot 10^6 \cdot \frac{lbf}{in^2}$

Equations:

$n := 2, 3 .. 10$

$$L := \frac{\frac{1}{3} \cdot \frac{OD}{2} + Len + \frac{1}{3} \cdot \frac{OD}{2}}{num + 1}$$

Mean Diameter $\quad D(t) := OD - t$

$$\rho(t,n) := \frac{1}{n^2 \cdot \left(\frac{2 \cdot L}{\pi \cdot D(t)}\right)^2 + 1}$$

$$\lambda2(t,n) := \rho(t,n) \cdot \left[3 + \mu + \left(1 - \mu^2\right) \cdot \rho(t,n)\right]$$

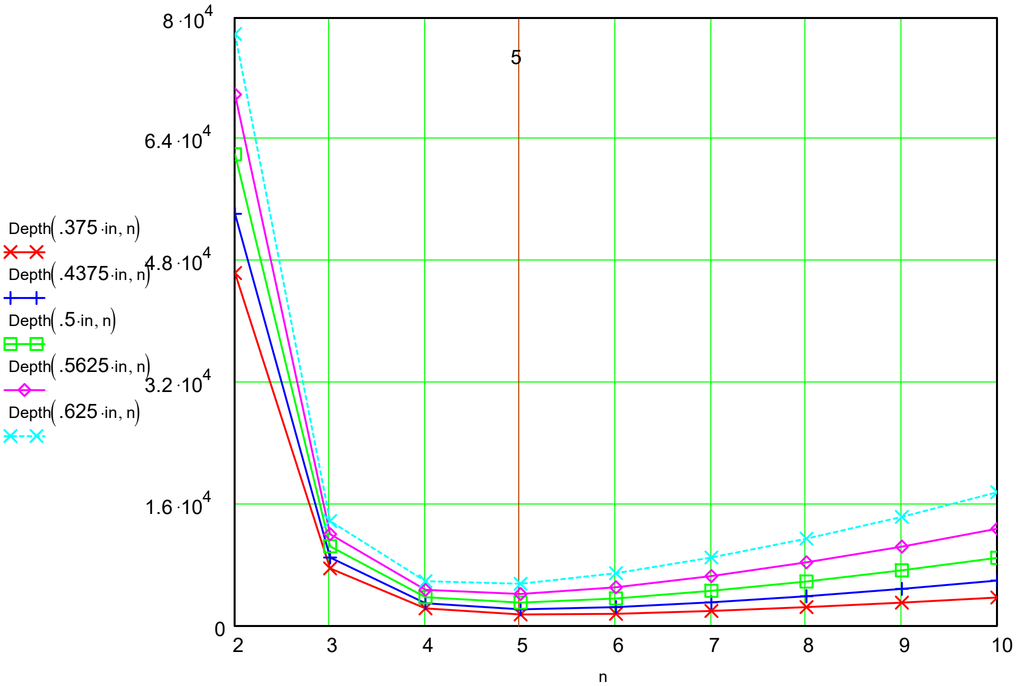
$$\lambda3(t,n) := \rho(t,n) \cdot (1 + \mu) - \left[\rho(t,n)^2 \cdot \left[\mu \cdot (1 + 2 \cdot \mu) + \left(1 - \mu^2\right) \cdot (1 - \rho(t,n) \cdot \mu) \cdot \left(1 + \frac{1 + \mu}{1 - \mu} \cdot \rho(t,n)\right)\right]\right]$$

$$\mu1(t,n) := 2 + \lambda2(t,n)$$

$$\mu2(t,n) := 1 + \lambda3(t,n)$$

$$FirstPart(t,n) := \frac{1}{3} \cdot \left[\left[n^2 + \left(\frac{\pi \cdot D(t)}{2 \cdot L}\right)^2\right]^2 - \mu1(t,n) \cdot n^2 + \mu2(t,n)\right] \cdot \frac{2 \cdot E}{\left(1 - \mu^2\right)} \cdot \left(\frac{t}{D(t)}\right)^3 + \frac{2 \cdot E \cdot \left(\frac{t}{D(t)}\right)}{\left[n^2 \cdot \left(\frac{2 \cdot L}{\pi \cdot D(t)}\right)^2 + 1\right]^2}$$

$$Depth(t,n) := FirstPart(t,n) \cdot \frac{1}{n^2 - 1 + \frac{1}{2} \cdot \left(\frac{\pi \cdot D(t)}{2 \cdot L}\right)^2} \cdot \frac{1}{SeaWaterDensity}$$



$$\frac{\text{Depth}(t,5)}{\text{ft}} = \begin{pmatrix} 1487 \\ 2167 \\ 3051 \\ 4169 \\ 5553 \end{pmatrix} \qquad \frac{t}{\text{in}} = \begin{pmatrix} 0.375 \\ 0.4375 \\ 0.5 \\ 0.5625 \\ 0.625 \end{pmatrix}$$

$OD := 39 \cdot in, 40 \cdot in .. 45 \cdot in$

$t := .5 \cdot in$

$n := 2, 3 .. 10$

$$L(OD) := \frac{\frac{1}{3} \cdot \frac{OD}{2} + Len + \frac{1}{3} \cdot \frac{OD}{2}}{num + 1} \quad * \qquad \text{Mean Diameter} \qquad D(OD) := OD - t$$

$$\rho(OD, n) := \frac{1}{n^2 \cdot \left(\frac{2 \cdot L(OD)}{\pi \cdot D(OD)}\right)^2 + 1} \quad *$$

$$\lambda 2(OD, n) := \rho(OD, n) \cdot \left[3 + \mu + \left(1 - \mu^2\right) \cdot \rho(OD, n)\right] \quad *$$

$$\lambda 3(OD, n) := \rho(OD, n) \cdot (1 + \mu) - \left[\rho(OD, n)^2 \cdot \left[\mu \cdot (1 + 2 \cdot \mu) + \left(1 - \mu^2\right) \cdot (1 - \rho(OD, n) \cdot \mu) \cdot \left(1 + \frac{1 + \mu}{1 - \mu} \cdot \rho(OD, n)\right)\right]\right] \quad *$$

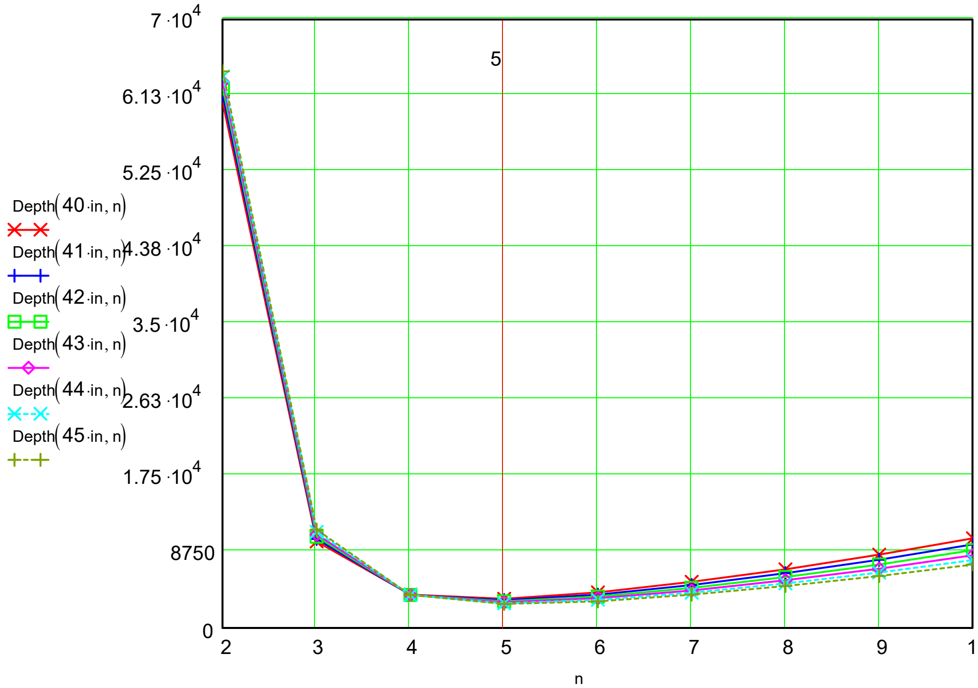
$$\mu 1(OD, n) := 2 + \lambda 2(OD, n) \quad *$$

$$\mu 2(OD, n) := 1 + \lambda 3(OD, n) \quad *$$

$$FirstPart(OD, n) := \frac{1}{3} \cdot \left[\left[n^2 + \left(\frac{\pi \cdot D(OD)}{2 \cdot L(OD)}\right)^2\right]^2 - \mu 1(OD, n) \cdot n^2 + \mu 2(OD, n)\right] \cdot \frac{2 \cdot E}{\left(1 - \mu^2\right)} \cdot \left(\frac{t}{D(OD)}\right)^3 \quad *$$

$$SecondPart(OD, n) := FirstPart(OD, n) + \frac{2 \cdot E \cdot \left(\frac{t}{D(OD)}\right)}{\left[n^2 \cdot \left(\frac{2 \cdot L(OD)}{\pi \cdot D(OD)}\right)^2 + 1\right]^2} \quad *$$

$$Depth(OD, n) := SecondPart(OD, n) \cdot \frac{1}{n^2 - 1 + \frac{1}{2} \cdot \left(\frac{\pi \cdot D(OD)}{2 \cdot L(OD)}\right)^2} \cdot \frac{1}{SeaWaterDensity} \quad *$$



$\frac{OD}{in} =$

39
40
41
42
43
44
45

$$\frac{\text{Depth}(OD, 5)}{ft} = \begin{pmatrix} 3516 \\ 3342 \\ 3188 \\ 3051 \\ 2930 \\ 2822 \\ 2727 \end{pmatrix}$$